Disc 104

Wednesday, February 24, 2021 11:01 AM

3.2 Problems

Problem 1. *Use Neville's method to approximate $\sqrt{3}$ with the following functions and values:*

1. $f(x) = 3^x$ *and the values* $x_0 = -2, x_1 = -1, x_2 = 0, x_3 = 1, x_4 = 2$
2. $f(x) = \sqrt{x}$ *and the values* $x_0 = 0, x_1 = 1, x_2 = 2, x_3 = 4, x_4 = 5$
3. *compare the accuracy of the approximations in parts (a) and (b)*

Problem 2. *Let $P_3(x)$ be the interpolating polynomial for the data $(0,0), (.5, y), (1,3), (2,2)$, find y if the coefficient of x^3 in $P_3(x)$ is 6.*

$P_3(x) = 6x^3 + ax^2 + bx$

$y = \frac{6}{2^3} + \frac{a}{2^2} + \frac{b}{2}$

$3 = 6 + a + b$
$2 = 6 \cdot 2^3 + 2^2 \cdot a + 2 \cdot b$

$\rightarrow$

$a + b = -3$
$4a + 2b = -46$
$-2a + 2b = -6$

$2a = -40$

$\boxed{a = -20 \quad b = 17}$

$\boxed{y = \frac{6}{8} + \frac{-20}{4} + \frac{17}{2}}$

Theorem 3.5 Let f be defined at $x_0, x_1, \ldots, x_k$ and let x_j and x_i be two distinct numbers in this set. Then

$$P(x) = \frac{(x - x_j)P_{0,1,\ldots,j-1,j+1,\ldots,k}(x) - (x - x_i)P_{0,1,\ldots,i-1,i+1,\ldots,k}(x)}{(x_i - x_j)}$$

is the kth Lagrange polynomial that interpolates f at the $k + 1$ points $x_0, x_1, \ldots, x_k$.

Compute   includes y.

$$P_3(x) = \frac{P_{0,1,2,3}(x)\,(x - x_1) - P_{1,2,3}(x)\,(x - x_0)}{(x_0 - x_1)}$$

3.3 Problems

Problem 3. *Use Newton forward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specfied value using each of the polynomials*

1. $f(.43)$ *if*

x_i	$f(x_i)$
0	1
.25	1.64872
.5	2.71828
.75	4.48169

Problem 4. *Show that the polynomial interpolating the following data has degree three.*

x	-2	-1	0	1	2	3
$f(x)$	1	4	11	16	13	-4

$$P_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \cdots + a_n(x - x_0)\cdots(x - x_{n-1}), \quad (3.5)$$

$$a_k = f[x_0, x_1, x_2, \ldots, x_k],$$

$$f[x_i] = f(x_i). \quad (3.7)$$

The remaining divided differences are defined recursively; the *first divided difference* of f with respect to x_i and x_{i+1} is denoted $f[x_i, x_{i+1}]$ and defined as

$$f[x_i, x_{i+1}] = \frac{f[x_{i+1}] - f[x_i]}{x_{i+1} - x_i}. \quad (3.8)$$

$h = \frac{1}{4}$ $\boxed{x = .43} = x_0 + sh \Rightarrow s = \frac{.43 - x_0}{h} = 1.72$

$$P_n(x) = f(x_0) + \sum_{k=1}^{n} \binom{s}{k} \Delta^k f(x_0)$$

$\binom{s}{k} := \frac{s(s-1)\cdots(s-k+1)}{k!}$

$f \quad s \in \mathbb{R}$

3) $P_1(x) = f(x_0) + \binom{s}{1}\Delta f(x_0)$

$P_2(x) = f(x_0) + \binom{s}{1}\Delta f(x_0) + \binom{s}{2}\Delta^2 f(x_0)$

$= P_1(x) + \binom{s}{2}\Delta^2 f(x_0)$

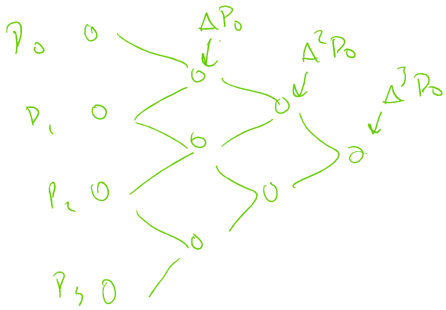
$P_3(x) = P_2(x) + \binom{s}{3}\Delta^3 f(x_0)$

$P_1(x) = 2.12$
$P_2(x) = 2.376$
$P_3(x) = 2.5606$

x_0			
1	(1)		0
1.6	1.07		
2.72	-		
4.5			

$\Delta P_n = P_{n+1} - P_n$

$\Delta^2(P_n) = \Delta(\Delta P_n) = \Delta P_{n+1} - \Delta P_n$



$$= P_{n+2} - P_{n+1} - (P_{n+1} - P_n)$$